# Nuclear energy – Quiz 2

## Solutions

## 14-10-2019

**1. Neutron poisons (12 pts)**

I-135 and Xe-135 are produced as fission fragments in a nuclear reactor. Assume that I-135 is only formed as the result of a fission event and that its microscopic absorption cross section is negligible. Xe-135 is produced directly by fission and indirectly by the decay of I-135. The half-lives of I-135 and Xe-135 are 6.57 h and 9.10 h, respectively.

$$ {}^{135}_{53}I \longrightarrow {}^{135}_{54}Xe \longrightarrow {}^{135}_{55}Cs $$

a. Write the equations for the rate of change in concentration for I-135 $\left(\frac{dI}{dt}\right)$ and Xe-135 $\left(\frac{dXe}{dt}\right)$. (3 pts)

**Solution**

$$ \frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I $$

$$ \frac{dXe}{dt} = \gamma_{Xe} \Sigma_f \phi + \lambda_I I - \lambda_{Xe} Xe - \sigma_a^{Xe}\, Xe\, \phi $$

b. The equilibrium concentration of I-135 is 2.6 $10^{15}$ atoms/cm$^3$. Give the expression of the I-135 concentration at equilibrium and determine the value of the neutron flux (in cm$^{-2}$.s$^{-1}$). (3 pts)

$\gamma_I$ = 0.0638
$\gamma_{Xe}$ = 0.0025
$\sigma_a^{Xe}$= 3.20*$10^{-18}$ cm$^2$
$\Sigma_f$ = 0.002 cm$^{-1}$

**Solution**

At equilibrium:

$$ \frac{dI}{dt} = 0 \Longrightarrow \gamma_I \Sigma_f \phi = \lambda_I I \Longrightarrow I = \frac{\gamma_I \Sigma_f \phi}{\lambda_I} \Longrightarrow \phi = \frac{\lambda_I I}{\gamma_I \Sigma_f} $$

$$\lambda_I = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{6.57 * 3600} = 2.93\ 10^{-5}\ s^{-1}$$

The neutron flux is:

$$\phi = \frac{2.93\ 10^{-5}\ s^{-1} \times 2.6\ 10^{15} cm^{-3}}{0.0638 \times 0.002\ cm^{-1}} = 5.97\ 10^{14}\ cm^{-2} \cdot s^{-1}$$

c. Determine the value of the xenon concentration at equilibrium. (2 pts)

**Solution**

At equilibrium:

$$\frac{dXe}{dt} = 0 \Longrightarrow \gamma_{Xe}\Sigma_f\phi + \lambda_I I = \lambda_{Xe}Xe + \sigma_a^{Xe}\ Xe\ \phi \Longrightarrow Xe = \frac{\gamma_{Xe}\ \Sigma_f^{fuel}\phi + \lambda_I\ I}{\lambda_{Xe} +\ \sigma_a^{Xe}\ \phi}$$

$$\lambda_{Xe} = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{9.1 * 3600} = 2.12\ 10^{-5}\ s^{-1}$$

$$Xe = \frac{0.0025 \times 0.002\ cm^{-1} \times 5.97\ 10^{14} cm^{-2} \cdot s^{-1} + 2.93\ 10^{-5}\ s^{-1} \times 2.6\ 10^{15} cm^{-3}}{2.12\ 10^{-5}\ s^{-1} + 3.2\ 10^{-18}\ cm^2 \times 5.97\ 10^{14}\ cm^{-2} \cdot s^{-1}}$$
$$= 4.1\ 10^{13}\ cm^{-3}$$

d. Show that under steady state conditions, Xe-135 is predominantly removed by burnout rather than by β-decay. (4 pts)

**Solution**

The rate at which xenon is removed by β-decay is:

$$\lambda_{Xe}Xe = 2.12\ 10^{-5}\ s^{-1} \times 4.1\ 10^{13}\ cm^{-3} = 8.69\ 10^{8}\ cm^{-3} \cdot s^{-1}$$

The rate at which xenon is removed by burnout is:

$$\sigma_a^{Xe}\ Xe\ \phi = 3.2\ 10^{-18}\ cm^2 \times 4.1\ 10^{13}\ cm^{-3} \times 5.97\ 10^{14}\ cm^{-2} \cdot s^{-1} = 7.83\ 10^{10} cm^{-3} \cdot s^{-1}$$

## 2. Reactivity in LWR (8 pts)

Consider a light-water reactor with the following characteristics:
$\varepsilon$ = 1.028; $L_{FNL}$= 0.886; f = 0.754; p = 0.813; $L_{TNL}$ = 0.955; $\eta$ = 2.021

a. A population of 15,000 neutrons exists at the beginning of a generation. Calculate the number of neutrons absorbed in the fuel and in the resonance peaks. (4 pts)

**Solution**

The multiplication factor is equal to

$$k = \frac{N}{N_0} = \varepsilon L_{FNL} p L_{TNL} f \eta$$

The number of neutrons absorbed in the fuel is

$$N = N_0 \varepsilon L_{FNL}\, p\, L_{TNL}\, f = 15000 \times 1.028 \times 0.886 \times 0.813 \times 0.955 \times 0.754 = 7998$$

Number of neutrons absorbed by the resonance peaks:

$$N = N_0 \varepsilon L_{FNL}(1-p) = 15000 \times 1.028 \times 0.886 \times (1-0.813) = 2554.8$$

b. What is the effect of an increase in the fuel's temperature on the factors of the 6- factor formula? How does such a change affect the reactivity? (4 pts)

**Solution**

Raising the temperature of the fuel causes the uranium nuclei to vibrate more rapidly within their lattice structures. Vibrating nuclei moving towards or away from the neutron can increase the absorption rate of the neutron with slightly lower or higher energy than the resonance one. Therefore, raising the temperature causes a broadening of the energy range of neutrons that may be resonantly absorbed in the fuel. The broadened resonances result in a larger percentage of neutrons having energies allowing their capture in the fuel pellets.

This results mainly in a reduction of the resonance escape probability $p$ and a decrease in reactivity.

**3. Diffusion of neutrons (10 pts)**

A critical and uniform spherical reactor with a diameter of 7.6 m has the following properties:
$\nu = 2.43; D = 1.01\ cm$
The material inside the reactor has an atomic mass of 270 and a density of 9.4 g/cm$^3$.
The average neutron flux is 2.1 $10^{12}$ cm$^{-2}$.s$^{-1}$ and the total absorption rate is 1.931 $10^{18}$ s$^{-1}$.
The Avogadro number is $N_A = 6.022\ 10^{23}\ mol^{-1}$.

a. Determine the values of the macroscopic and microscopic absorption cross sections. (4 pts)

**Solution**

The absorption reaction rate is given by

$$R = \Sigma_a \phi V = \Sigma_a \phi\ \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi\left(\frac{760}{2}\right)^3 = 2.3\ 10^8\ cm^3$$

The macroscopic cross section is

$$\Sigma_a = \frac{R}{\phi\, V} = \frac{1.931\ 10^{18}\ s^{-1}}{2.1\ 10^{12}\ cm^{-2}s^{-1} \times 2.3\ 10^8\ cm^3} = 0.0040\ cm^{-1}$$

The microscopic cross section is

$$\sigma_a = \frac{\Sigma_a}{N}$$

The number of nuclides in the reactor is

$$N = \frac{\rho \times N_A}{M} = \frac{9.4\ \frac{g}{cm^3} \times 6.022\ 10^{23}\ mol^{-1}}{270\ \frac{g}{mol}} = 2.09\ 10^{22}\ cm^{-3}$$

Therefore, the microscopic absorption cross section is

$$\sigma_a = \frac{0.0040\ cm^{-1}}{2.09\ 10^{22}\ cm^{-3}} = 1.91\ 10^{-25} cm^2 = 0.191\ barn$$

b. Calculate the buckling. (2 pts)

**Solution**

The buckling for a spherical reactor is given by

$$B^2 = \left(\frac{\pi}{r}\right)^2 = \left(\frac{\pi}{380}\right)^2 = 6.83\ 10^{-5}\ cm^{-2}$$

c. Determine the value of the macroscopic fission cross section. (2 pts)

**Solution**

The reactor is critical and the multiplication factor can be written as

$$k = 1 = \frac{\nu\, \Sigma_f}{\Sigma_a + DB^2}$$

Therefore, the macroscopic fission cross section is

$$\Sigma_f = \frac{\Sigma_a + DB^2}{\nu} = \frac{0.0040\ cm^{-1} + 1.01\ cm \times 6.83\ 10^{-5}\ cm^{-2}}{2.43} = 0.0017\ cm^{-1}$$

d. Calculate the total power in the reactor in MW considering that the energy released by one fission event is $E_f = 200\ MeV$. (2 pts)
1 MeV = 1.602 $10^{-13}$ J and 1 W = 1 J/s

**Solution**

The total power is equal to the average energy generated per fission × average fission rate × volume:

$$P = E_f \times \Sigma_f \phi \times V$$
$$= 200 MeV \times 1.602\ 10^{-13} J \times 0.0017\ cm^{-1} \times 2.1\ 10^{12}\ cm^{-2} s^{-1} \times 2.3\ 10^{8}\ cm^{3}$$
$$= 26.31\ MW$$

**Possibly useful equations**

$$N\left(\frac{atoms}{cm^3}\right) = \frac{\rho\left(\frac{g}{cm^3}\right) N_A\left(\frac{atoms}{mole}\right)}{M\left(u = \frac{g}{mole}\right)} \quad ; \quad \Sigma\ (cm^{-1}) = N\left(\frac{atoms}{cm^3}\right)\sigma(cm^2)$$

$$R_{reaction}\left(\frac{reactions}{cm^3 \times s}\right) = \phi\left(\frac{neutrons}{cm^2 \times s}\right) \times \Sigma_{reaction}(cm^{-1}) \quad ; \quad T_{1/2} = \frac{\ln 2}{\lambda}$$

$$k = \varepsilon L_{FNL} p L_{TNL} f \eta \quad ; \quad k_\infty = \frac{\nu \Sigma_f}{\Sigma_a} \quad ; \quad B^2 = \frac{\nu \Sigma_f - \Sigma_a}{D}$$

$$k = \frac{\nu \Sigma_f}{\Sigma_a + DB^2} \quad ; \quad P = E_f\, R_f V$$

| **Geometry** | **Dimensions** | **Buckling $B^2$** | **Flux** | **A** |
|---|---|---|---|---|
| Rectangular parallepiped | a x b x c | $\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$ | $A\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$ | $3.87 \times \frac{P}{V\ E_f\ \Sigma_f}$ |
| Finite cylinder | Radius R, height H | $\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$ | $AJ_0\left(\frac{2.405\ r}{R}\right)\cos\left(\frac{\pi\ z}{H}\right)$ | $3.63 \times \frac{P}{V\ E_f\ \Sigma_f}$ |
| Sphere | Radius R | $\left(\frac{\pi}{R}\right)^2$ | $A\ \frac{1}{r}\sin\left(\frac{\pi\ r}{R}\right)$ | $\frac{P}{4R^2\ E_f\ \Sigma_f}$ |